

# Pion-nucleon elastic scattering amplitude within covariant baryon chiral perturbation theory up to $O(p^4)$ level

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## Abstract

The  $O(p^4)$  calculation on pion-nucleon elastic scattering amplitude in EOMS scheme within covariant baryon chiral perturbation theory is reviewed. Numerical fits to partial wave amplitudes up to  $\sqrt{s} = 1.13\text{GeV}$  and  $1.20\text{GeV}$  are performed and the results are compared with previous studies.

**Keywords:**  $\pi$ - $N$  scattering, chiral perturbation theory, partial wave analysis

## 1. Introduction

Many efforts have been made in studying  $\pi$ - $N$  scatterings at low energies. However, unlike the successfulness of chiral perturbation theory in pure mesonic sector, a chiral expansion in  $\pi$ - $N$  scattering amplitude suffers from the power counting breaking (PCB) problem in the traditional subtraction  $\overline{MS} - 1$  scheme. [1] Many proposals have been made to treat this problem, e.g., heavy baryon chiral perturbation theory [2], infrared regularization scheme [3], extended on mass shell (EOMS) scheme [4], etc.. The EOMS scheme provides a good solution to the PCB problem, e.g., see [5], in the sense that it faithfully respects the analytic structure of the original amplitudes and being scale independent.

In this talk we will present our work on the  $O(p^3)$  and  $O(p^4)$  calculation on  $\pi$ - $N$  scattering amplitude in EOMS scheme and will compare it with previous results in the literature.

## 2. NNLO and NNNLO calculations

We start from the following effective lagrangian at  $O(p^3)$  level (extendable to  $O(p^4)$  [6]):

$$\mathcal{L}_{eff} = \bar{N} \left\{ i \not{D} - m + \frac{g_A}{2} \not{u} \gamma_5 + c_i O_i^{(2)} + d_j O_j^{(3)} \right\} N + \frac{f_\pi^2}{4} \langle u^\mu u_\mu + \chi_+ \rangle + \frac{\ell_4}{8} \langle u^\mu u_\mu \rangle \langle \chi_+ \rangle + \frac{\ell_3 + \ell_4}{16} \langle \chi_+ \rangle^2,$$

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where  $O^{(2)}$  and  $O^{(3)}$  are relevant operators of  $O(p^2)$  and  $O(p^3)$  respectively,  $i \in (1, 2, 3, 4)$  and  $j \in (1, 2, 3, 5, 14, 15, 16, 18)$  [6].

Decomposition of  $\pi$ - $N$  amplitude is standard,

$$\begin{aligned} T_{\pi N}^{a'a} &= \delta_{a'a} T^+ + \frac{1}{2} [\tau_{a'}, \tau_a] T^-, \\ T^\pm &= \bar{u}(p', s') \left[ D^\pm + \frac{i}{2m_N} \sigma^{\mu\nu} q'_\mu q_\nu B^\pm \right] u(p, s). \end{aligned} \quad (1)$$

To carry out the calculation in EOMS scheme one firstly perform  $\overline{MS} - 1$  subtraction to remove ultraviolet divergencies, then additional subtraction (A.S.) to absorb PCB terms. Taking the nucleon mass renormalization for example, one has,

$$\begin{aligned} m_N &= m - 4c_1 M^2 - \frac{3mg^2}{2f^2} [\Delta_N - M^2 I(m^2)] \\ &= \hat{m} - 4c_1^r M^2 + \frac{3mM^2 g^2}{2f^2} \bar{I}(m^2) \quad (\overline{MS} - 1) \\ &= \hat{m} - 4\tilde{c}_1^r M^2 + \frac{3mM^2 g^2}{2f^2} \bar{I}(m^2) - \frac{3mM^2 g^2}{32\pi^2 f^2} (\text{A.S.}), \end{aligned} \quad (2)$$

where  $\hat{m}$  is the nucleon mass in chiral limit. The last term on the *r.h.s.* of the third equality is opposite to the PCB term which is absorbed by redefining  $c_1^r$  as:  $\tilde{c}_1^r = c_1^r - \frac{3g^2 m}{128f^2 \pi^2}$ . Definitions of all functions appeared here follow from Appendix A.

Another example is the calculation of the axial-vector coupling  $g_A$ :

$$\begin{aligned} g_A &= g + 4d_{16} M^2 - \frac{g^3 m^2}{32f^2 \pi^2} + \frac{g(4 - g^2)}{2f^2} \Delta_N - \frac{g(2 + g^2)}{2f^2} \Delta_\pi \\ &+ \frac{g^3(2m^2 + M^2)}{4f^2} J_N(0) - \frac{g(8 - g^2)M^2}{4f^2} I(m^2) - \frac{g^2 M^4}{4f^2} I_A(0) \end{aligned}$$

Table 1: Fitting results at  $O(p^3)$  and  $O(p^4)$ . Given for comparison are results from [7, 9]. The  $c_i$ ,  $d_j$  and  $e_k$  have, respectively, units of  $\text{GeV}^{-1}$ ,  $\text{GeV}^{-2}$  and  $\text{GeV}^{-3}$ . In  $O(p^4)$  fits, the fitted  $c_i$  here should be understood as  $\hat{c}_i$ :  $\hat{c}_1 = c_1 - 2M^2(e_{22} - 4e_{38})$ ,  $\hat{c}_2 = c_2 + 8M^2(e_{20} + e_{35})$ ,  $\hat{c}_3 = c_3 + 4M^2(2e_{19} - e_{22} - e_{36})$ ,  $\hat{c}_4 = c_4 + 4M^2(2e_{21} - e_{37})$ .

LEC	Fit I- $O(p^3)$	Ref. [7]- $O(p^3)$	Fit II- $O(p^3)$	Ref. [9]- $O(p^3)$	Fit I- $O(p^4)$	Fit II- $O(p^4)$
$c_1$	$-1.39 \pm 0.06$	$-1.50 \pm 0.06$	$-0.81 \pm 0.03$	$-1.00 \pm 0.04$	$-1.09 \pm 0.06$	$-0.98 \pm 0.03$
$c_2$	$4.00 \pm 0.09$	$3.74 \pm 0.09$	$1.46 \pm 0.09$	$1.01 \pm 0.04$	$2.79 \pm 0.10$	$1.41 \pm 0.04$
$c_3$	$-6.59 \pm 0.08$	$-6.63 \pm 0.08$	$-3.09 \pm 0.12$	$-3.04 \pm 0.02$	$-5.32 \pm 0.14$	$-3.76 \pm 0.04$
$c_4$	$3.91 \pm 0.04$	$3.68 \pm 0.05$	$2.35 \pm 0.06$	$2.02 \pm 0.01$	$2.38 \pm 0.19$	$1.16 \pm 0.03$
$d_1 + d_2$	$4.32 \pm 0.53$	$3.67 \pm 0.54$	$0.78 \pm 0.09$		$6.21 \pm 0.12$	$2.14 \pm 0.04$
$d_3$	$-3.00 \pm 0.50$	$-2.63 \pm 0.51$	$-0.46 \pm 0.05$		$-6.86 \pm 0.16$	$-3.88 \pm 0.05$
$d_5$	$-0.56 \pm 0.13$	$-0.07 \pm 0.13$	$-0.16 \pm 0.04$		$0.54 \pm 0.11$	$1.17 \pm 0.04$
$d_{14} - d_{15}$	$-7.05 \pm 1.05$	$-6.80 \pm 1.07$	$-0.89 \pm 0.15$		$-11.90 \pm 0.24$	$-3.96 \pm 0.08$
$d_{18}$	$-0.74 \pm 1.41$	$-0.50 \pm 1.43$	$-0.92 \pm 0.25$		$-0.74(\text{input})$	$-0.74(\text{input})$
$e_{14}$	-	-	-	-	$3.68 \pm 0.36$	$2.62 \pm 0.09$
$e_{15}$	-	-	-	-	$-14.67 \pm 0.55$	$-5.15 \pm 0.13$
$e_{16}$	-	-	-	-	$7.15 \pm 0.35$	$1.55 \pm 0.07$
$e_{17}$	-	-	-	-	$0.57 \pm 1.34$	$13.57 \pm 0.15$
$e_{18}$	-	-	-	-	$3.64 \pm 1.18$	$-9.05 \pm 0.12$
$h_A$	-	-	$2.82 \pm 0.04$	$2.87 \pm 0.04$	-	$2.82(\text{input})$
$\chi^2_{d.o.f}$	0.18	0.22	0.35	0.23	0.04	0.21

$$\begin{aligned}
& + \frac{3g^3m^2M^2}{f^2} \frac{\partial I(s)}{\partial s} I(s) \Big|_{p=m_N} \\
& = \hat{g}_A + 4d_{16}M^2 - \frac{g(2+g^2)}{2f^2} \Delta_\pi + \frac{3g^3m^2M^2}{f^2} \frac{\partial I(s)}{\partial s} \Big|_{p=m_N} \\
& + \frac{g^3M^2}{4f^2} J_N(0) - \frac{g(8-g^2)M^2}{4f^2} I(m^2) - \frac{g^2M^4}{4f^2} I_A(0), \quad (3)
\end{aligned}$$

where  $\hat{g}_A$  is the axial charge in the chiral limit. Ultraviolet divergencies are treated by  $\overline{MS} - 1$  subtraction. If we start with  $\hat{g}_A$ , there are no PCB terms to be extracted. The PCB effects are included in  $\hat{g}_A$ . If we start with a bare  $g$ , we need to redefine it as,  $\tilde{g} = g^r - \frac{g^3m^2}{16f^2\pi^2}$ ,  $g^r = g + \frac{g(2-g^2)m^2}{16f^2\pi^2}R$ . We prefer the latter hereafter, i.e. starting with bare parameters.

Similar to  $m_N$  and  $g_A$  renormalization, the calculation of scattering amplitude up to  $O(p^3)$  in EOMS scheme is straightforward, if the PCB terms in functions  $D$  and  $B$  for loop amplitudes are known,

$$\begin{aligned}
D_{PCB}^+ &= \frac{1}{64f^4m\pi^2\sigma^2} \left\{ 6g^2m^2M^2\sigma^2 + 2\sigma^4 \right. \\
&\quad \left. + g^4 \left[ 2m^4 (10M^4 - 7M^2t + t^2) \right. \right. \\
&\quad \left. \left. + 3m^2 (3t - 7M^2)\sigma^2 + \sigma^4 \right] \right\}, \\
D_{PCB}^- &= \frac{g^4m}{64f^4\pi^2\sigma^2} \left\{ \sigma^2 (t - 2M^2 + 2\sigma) \right. \\
&\quad \left. - 2m^2 (2M^2 - t) (2M^2 - t + 2\sigma) \right\},
\end{aligned}$$

$$\begin{aligned}
B_{PCB}^+ &= \frac{g^4m^4}{8f^4\pi^2\sigma^2} (2M^2 - t + 2\sigma), \\
B_{PCB}^- &= \frac{g^2m^2}{32f^4\pi^2\sigma^2} \left\{ 5\sigma^2 + g^2 \left[ 4m^2(t - 5M^2) + 3\sigma^2 \right] \right\} \quad (4)
\end{aligned}$$

where  $\sigma = s - m^2$ . After mass and  $g_A$  renormalization, the PCB terms above can be absorbed by redefining  $c_i^r$ s:

$$\begin{aligned}
c_1^r &\rightarrow \tilde{c}_1 = c_1^r - \frac{3g^2m}{128F^2\pi^2} \\
c_2^r &\rightarrow \tilde{c}_2 = c_2^r + \frac{(2+g^4)m}{32f^2\pi^2}, \\
c_3^r &\rightarrow \tilde{c}_3 = c_3^r - \frac{9g^4m}{64f^2\pi^2}, \\
c_4^r &\rightarrow \tilde{c}_4 = c_4^r + \frac{g^2(5+g^2)m}{64f^2\pi^2}, \quad (5)
\end{aligned}$$

and the  $\tilde{c}_i^r$ s are determined by fitting data. Theoretically, the NNLO amplitudes keep good analytic, correct power counting and scale-independent properties.

In the following we further extend the above calculation to  $O(p^4)$  level:

$$\begin{aligned}
m_N &= m + \cdots - 2(8e_{38} + e_{115} + e_{116})M^4 \\
&\quad + \frac{3M^2\Delta_\pi}{f^2} \left[ (2c_1 - c_3) - \frac{c_2}{d} \right], \quad (6) \\
g_A &= g + \cdots - \frac{2g}{mf^2} \left\{ c_2 \left( \frac{4M^2\Delta_\pi + m^2\Delta_N}{d} - M^2I^{(2)}(m^2) \right) \right\}
\end{aligned}$$

$$-4m^2 \left[ (c_3 + c_4) I^{(2)}(m^2) + c_4 (\Delta_\pi - M^2 I(m^2)) \right] \}. \quad (7)$$

Only  $O(p^4)$  parts are shown explicitly on the *r.h.s.* of Eqs. (6), (7), and ellipses represent lower order contributions given by Eqs. (2), (3). It is worth noticing that when obtaining the  $O(p^4)$  results, replacement of  $m$  in nucleon propagator with  $m_2 = m - 4c_1 M^2$ , namely making Dyson resummation to renormalize  $m$  to  $m_2$  first, will simplify calculations greatly [3]. The  $O(p^4)$  part in Eq. (6) doesn't contribute PCB terms, while the one in Eq. (7) does and  $g^r$  is now redefined as  $\tilde{g} = g^r - \frac{g^3 m^2}{16f^2 \pi^2} + \frac{gm^3}{576f^2 \pi^2} (9c_2 + 32c_3 + 32c_4)$ .

PCB terms of the fourth-order loop amplitude read,

$$\begin{aligned} B_{PCB}^+ &= \frac{-m}{576f^4 \pi^2 \sigma^3} \left\{ [24c_4 + (67c_2 - 56c_3 + 96c_4)g^2] \sigma^4 \right. \\ &\quad + 32(2c_2 + 17c_3 - 19c_4)g^2 m^2 M^2 \sigma^2 \\ &\quad + 2(9c_2 + 32c_3 + 32c_4)g^2 m^4 \\ &\quad \times \left[ 4M^4 \sigma + t^2 - t + 2\sigma^2 + M^2(-4t + 2\sigma) \right] \}, \\ B_{PCB}^- &= \frac{m^3}{576f^4 \pi^2 \sigma^3} \left\{ (9c_2 + 32c_3 + 16c_4) \sigma^3 \right. \\ &\quad - 2(9c_2 + 16c_3 - 28c_4)g^2 \sigma^3 \\ &\quad + 2g^2 m^2 (9c_2 + 32c_3 + 32c_4) \\ &\quad \times (2M^2 - t)(2M^2 - t + \sigma) \}, \end{aligned} \quad (8)$$

and  $D_{PCB}^\pm$  terms as well as the full amplitude are also obtained but are very lengthy, so we will present it elsewhere. [10]

### 3. Numerical studies and conclusions

At  $O(p^3)$  level we have performed two fits, the first one is up to  $\sqrt{s} = 1.13\text{GeV}$ , the second is up to  $\sqrt{s} = 1.20\text{GeV}$  for the convenience of comparing with the numerical studies given in Ref. [7–9]. Data being fitted are from Ref. [11] and error are assigned with the method of Ref. [8]. For the second fit we also included the tree level  $\Delta(1232)$  contribution [12], characterized by the  $N\Delta$  axial coupling  $h_A$ . Fit results are summarized in Table 1, where we have also listed the results from Refs. [7] and [9] for comparison. We see that, in general, our fit results at  $O(p^3)$  level are in good agreement with that of Refs. [7, 9], except the  $d_5$  parameter. We also listed our  $O(p^4)$  results from the best solution in our fits. To let the fitted LECs same as [13],  $d_{18}$  and  $h_A$  are fixed at their  $O(p^3)$  fitting results. In Figures 1 and 2 we plot the fit up to  $\sqrt{s} = 1.13\text{GeV}$  and  $1.20\text{GeV}$ , respectively. We find that, both  $O(p^3)$  and  $O(p^4)$  calculations give a reasonable description to data and the  $O(p^4)$  calculation improves the fit quality.

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### Appendix A. Definition of loop integrals

- 1 meson:  $\Delta_\pi = I_{10}$   

$$\Delta_\pi = \frac{1}{i} \int \frac{d^d k}{(2\pi)^d} \frac{1}{M^2 - k^2}.$$
- 1 nucleon:  $\Delta_N = I_{01}$   

$$\Delta_N = \frac{1}{i} \int \frac{d^d k}{(2\pi)^d} \frac{1}{m^2 - k^2}.$$
- 1 meson, 1 nucleon:  $I = I_{11}$   

$$\{I, I^\mu, I^{\mu\nu}\} = \frac{1}{i} \int \frac{d^d k}{(2\pi)^d} \frac{\{1, k^\nu, k^\mu k^\nu\}}{[M^2 - k^2][m^2 - (\Sigma - k)^2]},$$

$$I^\mu(s) = \Sigma^\mu I^{(1)}(s),$$

$$I^{\mu\nu}(s) = g^{\mu\nu} I^{(2)}(s) + \Sigma^\mu \Sigma^\nu I^{(3)}(s).$$
- 2 nucleons:  $J_N = I_{02}$   

$$J_N = \frac{1}{i} \int \frac{d^d k}{(2\pi)^d} \frac{1}{[m^2 - (k - P)^2][m^2 - (k - P')^2]}.$$
- 1 mesons, 2 nucleon:  $I_A$   

$$I_A = \frac{1}{i} \int \frac{d^d k}{(2\pi)^d} \frac{1}{[M^2 - k^2][m^2 - (P - k)^2][m^2 - (P' - k)^2]}.$$

After removing part proportional to  $R = -\frac{1}{\epsilon} + \gamma_E - 1 - \ln 4\pi$ , the remaining scalar integrals are finite and denoted by, e.g.  $\bar{I}(s)$ ,  $\bar{J}_N(t)$ ,  $\bar{I}_A(t)$ , etc..

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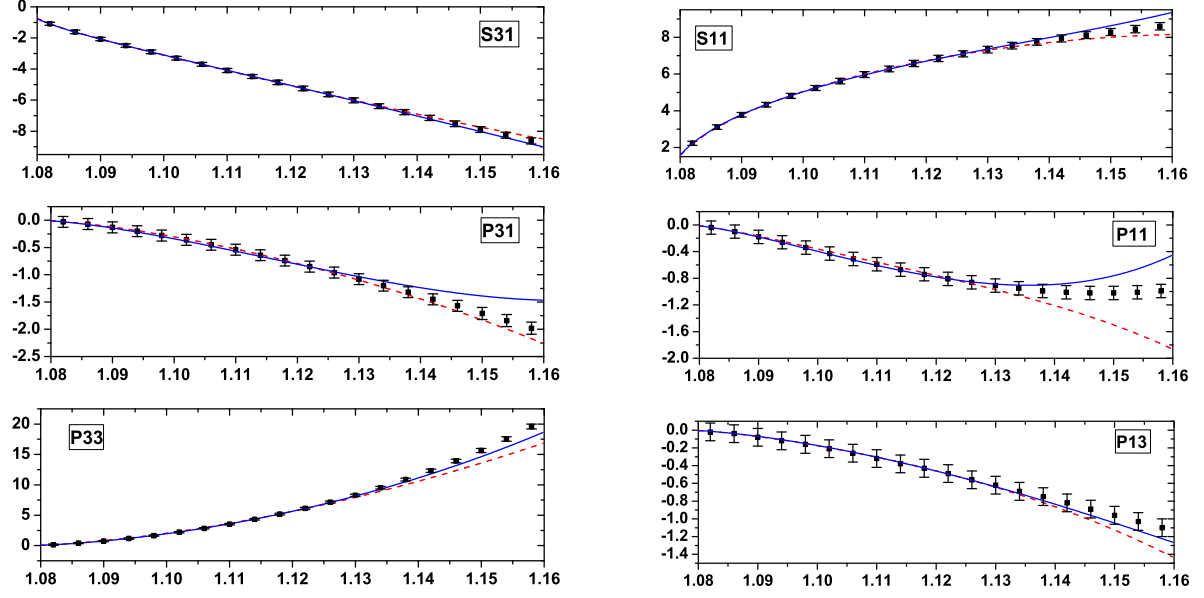


Figure 1: (Color online) Fit up to 1.13 GeV. The fourth- and third-order fits are presented by the solid(blue) and dash(red) lines respectively.

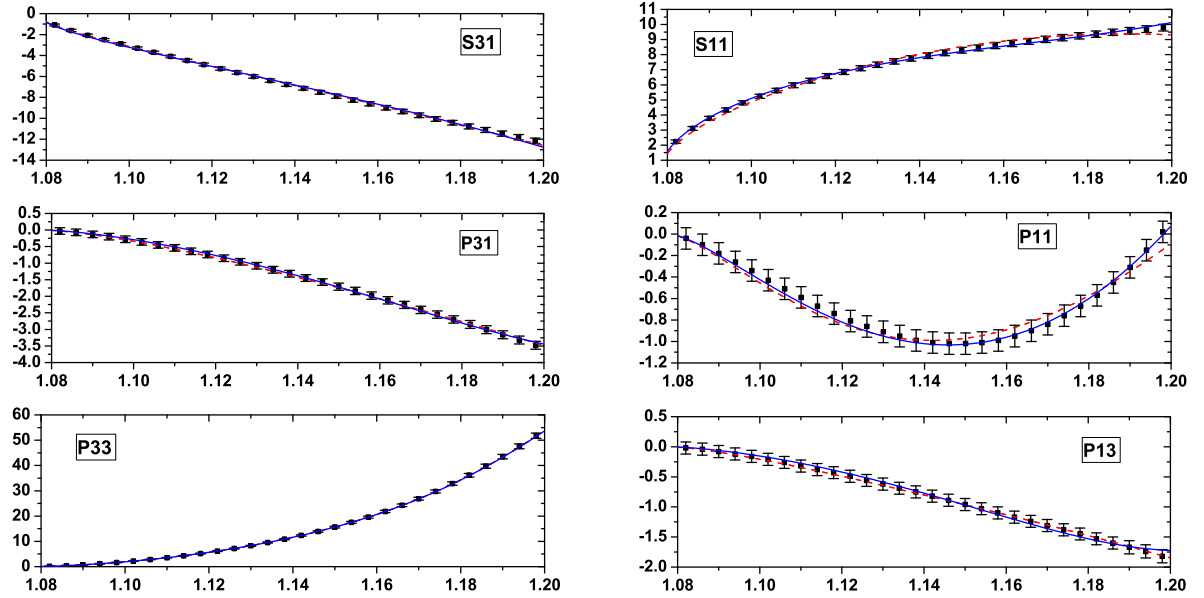


Figure 2: (Color online) Fit up to 1.20 GeV. The fourth- and third-order fits are presented by the solid(blue) and dash(red) lines respectively.